PERGAMON

International Journal of Heat and Mass Transfer 44 (2001) 2827–2833

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Equipartition of forces as a lower bound on the entropy production in heat exchange

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Received 22 November 1999; received in revised form 12 October 2000

Abstract

We show that the optimum operation of one heat exchanger is the operation that is known since long from experience, namely the one given by certain counter-current flows. The result is according to the principle of equipartition of forces (EoF). The principle of equipartition of forces poses ideal boundary conditions for operation. The boundary conditions may not be realizable in practice, but give a lower bound on the entropy production of the heat exchange process, and thus a measure on the efficiency of the process. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

Bejan [3] has discussed the design of a wide variety of heat exchangers that operate with minimum entropy production or maximum second law efficiency. This work is also concerned with heat exchange at minimum entropy production. We shall not, however, find the entropy production for *given boundary conditions,* as Bejan [3] does. We shall *determine* (ideal) boundary conditions that are compatible with a state of minimum entropy production for a given duty.

Our problem must not be confused with the typical industrial problem; that is to find the minimum area of heat exchange by varying the flow rate and the outlet temperature of the coolant. This question has its direct answer by solving the energy balance. We shall instead first find ideal boundary conditions from the second law of thermodynamics. The energy balance is next applied in order to realize the ideal conditions. It is of interest to study the ideal heat exchange process, to see how far it is possible to increase the second law efficiency of heat exchange.

The problem we raise, was solved on general grounds by Sauar et al. [14]. Minimum entropy production for a process with a given production (duty), was obtained with a constant driving force. The force was defined by irreversible thermodynamics. A constant driving force gives a lower bound on the process, and enables us to measure the distance of any real process to the most efficient one. No restriction was found for the transport coefficient [14] provided that the optimization problem was formulated according to the rules of irreversible thermodynamics. The constant force represents the boundary conditions that we are looking for. The proof has not yet been applied to heat exchangers, which is of interest here.

Some questions related to the proof arise from the literature. Tondeur and Kvaalen [18], Tondeur [17] stated that the local entropy production rate is constant for heat exchange. Their force for heat transfer was $\Delta(1/T)$. With a constant heat transfer coefficient, this is equivalent to a constant force. It has been assumed that the thermal conductivity must be constant in order for the optimal driving force to be constant [17]. Haug-Warberg [6] has recently argued the same. On the other hand, Bejan [1,2], and Minta and Smith [9] in their construction of a helium liquefaction cycle found that an optimal cycle was obtained when the temperature difference between the media, over the average temperature, $\Delta T/T$, was everywhere constant. In a thermal design study of LNG heat exchangers, Fredheim [5] found that the exergy loss in the heat exchanger was minimum when the temperature difference between the heating and the cooling medium was constant. Minimum

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Nomenclature

A	Total heat exchange area (m^2)
A_{co}	Total heat exchange area in the co-current case (m^2)
$A_{counter}$	Total heat exchange area in the counter-current case (m^2)
A_n	Heat exchange area of control volume n (m^2)
h	Enthalpy (J/kg)
J_i	Flux
J'_q	Measurable heat flux (J/m^2 s)
$J'_{q,n}$	Measurable heat flux through control volume n (J/m^2 s)
J_s	Entropy flux (J/m^2 K)
l_{qq}	Phenomenological heat transfer coefficient (J K/m s)
$\bar{l}_{qq}$	Average phenomenological heat transfer coefficient (J K/m^2 s)
$\bar{l}_{ss}$	Average phenomenological entropy transfer coefficient (J/m^2 K s)
mC_p	Product of mass flow and heat capacity (J/K s)
n	Control volume
$\boldsymbol{n}_c$	Mass flux of the cold fluid (kg/m^2 s)
$\boldsymbol{n}_h$	Mass flux of the hot fluid (kg/m^2 s)
p	Number of control volumes
Q	Heat exchanger duty (J/s)
T	Temperature (K)
T_c	Temperature of the cold fluid (K)
T_{cl}	Temperature of the cold fluid on the left side of the heat exchanger (K)
T_{cr}	Temperature of the cold fluid on the right side of the heat exchanger (K)
T_h	Temperature of the hot fluid (K)
T_{hl}	Temperature of the hot fluid on the left side of the heat exchanger (K)
T_{hr}	Temperature of the hot fluid on the right side of the heat exchanger (K)
$\Delta(1/T)_l$	Inverse temperature difference at the left side of the unit (1/K)
$\Delta(1/T)_n$	Inverse temperature difference across control volume n (1/K)
$\Delta(1/T)_{opt}$	Optimal inverse temperature difference (1/K)
$\Delta(1/T)_r$	Inverse temperature difference at the right side of the unit (1/K)
U	Heat transfer coefficient as used in the chemical engineering literature (W/m^2 K)
X_i	Force conjugate to J_i

Greek symbols

Δ	Difference
δ	Thickness of heat exchange medium including the liquid and gas film (m)
λ	Lagrange multiplier when transferred heat is used as constraint
λ_s	Lagrange multiplier when entropy flow is used as constraint
Σ	Total entropy production (J/K s)
σ	Local entropy production (J/K s m^3)

exergy loss is equivalent to minimum entropy production. The first question that arises from the literature is therefore: If a constant driving force is characteristic for a maximum efficiency, how is this force defined? We shall furthermore see in more detail that the coefficient need not be constant for the constant force criterion to be true, and that the force is defined completely within the framework of irreversible thermodynamics once a choice is made for the flux.

2. Theory

2.1. The system

The system consists of a simple heat exchanger with two fluids separated by a thin metal plate. A sketch of the unit and an enlarged control volume is given in Fig. 1. The heat exchanger might for instance cool a hydrocarbon oil with water. The fluids flow at constant rates in the z-direction and we assume plug flow with perfect thermal mixing in the x- and y-directions. The

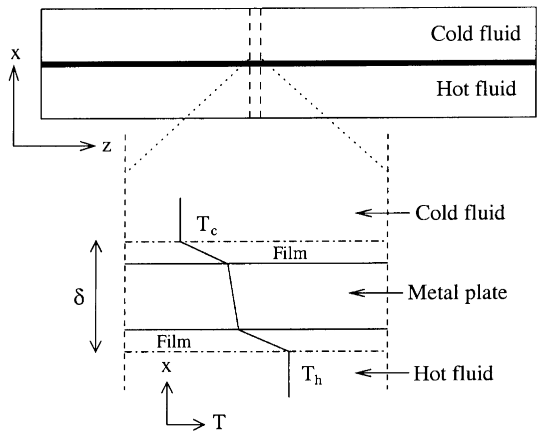

Fig. 1. A schematic representation of the heat exchanger and an enlarged control volume.

fluids can pass each other in co- or counter-current fashions. The heat flux is directed from one fluid to the other; in the x-direction, normal to the metal plate. It is generated by temperature differences across the two metal-fluid films and the metal plate. We assume that heat is transferred by convection between the fluid films and the metal plate, and by conduction inside the metal. In the fluids and the metal, we neglect heat conduction in the y- and z-directions.

We shall deal with stationary states only. For a stationary state heat flux in the x-direction in the fluid films and the metal plate, we have

$$\frac{\mathrm{d}}{\mathrm{d}x} J'_q(y,z) = 0. \tag{1}$$

This means that the measurable heat flux through the films and the metal, J'_q, is constant with x. It can vary with y and z.

2.2. The optimization problem

We want to minimize the total entropy production (Σ) of the system at a given duty (Q). The total entropy production is obtained by integrating the local entropy production (σ) over the volume of the wall separating the two fluids. In general, the local entropy production is a sum of products of fluxes and their corresponding (conjugate) forces [4,11,12]

$$\sigma = \sum_i J_i X_i. \tag{2}$$

Here J_i is a flux and X_i its conjugate force. The flux can be a vector, e.g. heat flux, or a scalar, e.g. reaction rate. We shall assume that the entropy production due to heat transfer can vary, while frictional losses due to fluid flow and turbulence are constant. Eq. (2) then simplifies to

$$\sigma = J'_q(y,z) \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{T}\right), \tag{3}$$

where T is the absolute temperature and $\mathrm{d}/\mathrm{d}x(1/T)$ is the force conjugate to J'_q.

We integrate Eq. (3) over the thickness of the fluid films and the metal plate (δ) to find the entropy production in a control volume of infinitesimal thickness dy and dz

$$\int_\delta \sigma \, \mathrm{d}x = J'_q(y,z)\Delta\left(\frac{1}{T}\right). \tag{4}$$

On the basis of the integrated form, we can write the measurable heat flux proportional to its conjugate force

$$J'_q(y,z) = \bar{l}_{qq}\Delta\left(\frac{1}{T}\right), \tag{5}$$

where $\bar{l}_{qq}$ is the average coefficient for heat transfer in the x-direction. The coefficient $\bar{l}_{qq}$ can vary with the properties of the liquid film, the type of material used, and the local temperature, but according to the theory, $\bar{l}_{qq}$ is independent of the force. In the present context this means the difference of the inverse temperatures on the two sides. The assumption of the coefficient being independent of the force on a local level, has been confirmed for all practical purposes by Nettleton [10]. In the following, we shall write the force as a function of the flux and generalize the notation somewhat

$$\Delta\left(\frac{1}{T}\right) \equiv X(y,z) = R(y,z)J'_q(y,z). \tag{6}$$

The duty of the heat exchanger is the integral of $J'_q(y,z)$ over the heat exchange area, A

$$Q = \int_A J'_q(y,z)\, \mathrm{d}A. \tag{7}$$

The total entropy production of the heat exchanger is therefore

$$\Sigma = \int_A \int_\delta \sigma \, \mathrm{d}x \, \mathrm{d}A = \int_A J'_q(y,z)\Delta\left(\frac{1}{T}\right) \mathrm{d}A. \tag{8}$$

We can now carry out the minimization of Eq. (8) at the duty given by Eq. (7). By using the Euler–Lagrange method, we obtain

$$\frac{\delta}{\delta J'_q(y,z)}[\Sigma + \lambda Q] = \frac{\delta}{\delta J'_q(y,z)} \int_A [R(y',z')J'^2_q(y',z') + \lambda J'_q(y',z')]\, \mathrm{d}y' \, \mathrm{d}z' = 0, \tag{9}$$

where δ means functional derivative. We have introduced the linear relation (6). Since we assume that heat is transferred only in the x-direction in the control volumes, i.e., they do not exchange heat (see Eq. (1)), we can simplify the above expression:

$$\frac{\delta}{\delta J'_q(y,z)}[R(y,z)J'^2_q(y,z) + \lambda J'_q(y,z)] = 2R(y,z)J'_q(y,z) + \lambda = 0. \tag{10}$$

Since $\bar{l}_{qq}$ or $R(y,z)$ are independent of the force, we find the derivatives in a straightforward manner. The optimal thermal force that gives the lowest entropy production at a given duty, is

$$X(y,z) = \Delta\left(\frac{1}{T}\right)_{\mathrm{opt}} = -\frac{\lambda}{2}. \tag{11}$$

Eq. (11) states that the difference in inverse temperatures is constant at minimal entropy production throughout the entire heat exchanger. This result was called the principle of equipartition of forces (EoF) [14]. The result is true *regardless* of the value of $\bar{l}_{qq}(y,z)$, or the thickness of the wall being constant or not, provided that the transport paths are parallel. By using the constant force, we can rewrite Eq. (7) by combining it with Eq. (5)

$$\Delta\left(\frac{1}{T}\right)_{\mathrm{opt}} = \frac{Q}{\int_A \bar{l}_{qq}\, \mathrm{d}A}. \tag{12}$$

This equation enables us to calculate the optimal driving force for the heat exchange process.

Experimental data for heat transfer are normally fitted to Fourier's law on integrated form

$$J'_q = -U\Delta T, \qquad (13)$$

where U is the common heat transfer coefficient and $\Delta T = T_c - T_h$. It is common to assume that U is constant, but it is known that it is a function of the flow pattern (fluid films), the temperature and the heat transfer medium. Both U and $\bar{l}_{qq}$ are found from experimental data, and are as such not exact mathematical functions. The same set of data can equally well be fitted to Eq. (5). A numerical *estimate* for $\bar{l}_{qq}$ that makes us able to use published values for U in a certain limited temperature interval, is

$$\bar{l}_{qq} = UT_h^2. \qquad (14)$$

This estimate does not make the coefficient dependent on the force. The coefficient $\bar{l}_{qq}$ varies with temperature, but this does not alter the result of the optimization, as discussed above.

So far we have focused on the measurable heat flux and its driving force. Irreversible thermodynamics offer alternative equivalent choices for the thermal flux-force pair. Alternatively, we may choose the entropy flux, J_s, as the flux of interest, as was done by Minta and Smith [9]. This choice of flux must be used when the constraint is on the total entropy that must be transferred. The entropy production is alternatively, with $J_s = J'_q/T$,

$$\sigma = -J_s \frac{\partial \ln T}{\partial x}.$$

A linear relation can also be written between J_s and $-\partial \ln T/\partial x$. Following the same reasoning as above we arrive at analogous results for the design criterion

$$-\Delta(\ln T)_{\mathrm{opt}} = -\left(\frac{\Delta T}{T}\right) = \frac{Q}{\int_A \bar{l}_{ss}\,\mathrm{d}A} = -\frac{\lambda_s}{2}, \qquad (15)$$

and the average coefficient of transfer

$$\bar{l}_{ss} = -\frac{J_s}{\Delta(\ln T)}. \qquad (16)$$

The coefficient $\bar{l}_{ss}$ has a different temperature dependence than that of $\bar{l}_{qq}$.

3. Calculations

Four different cases of heat exchange were calculated, all at constant $Q, U, \boldsymbol{n}_c, \boldsymbol{n}_h$ (mass flows), T_{hl} and T_{hr}. The subscripts c and h mean cold and hot fluid, while hl and hr refer to the left and right side of the hot fluid, respectively (see Fig. 1). The corresponding temperatures for the cold fluid were T_{cl} and T_{cr}.

The following numerical data were chosen to demonstrate typical effects. The duty of the heat exchanger was $Q = 60$ kW, the heat transfer coefficient was $U = 340$ W/m^2 K, the mass flow of the hot stream was $\boldsymbol{n}_h = 1$ kg/s, and the heat capacity was 2 kJ/kg K. The heat capacity for the coolant (water) was 4.2 kJ/kg K and the mass flow was $\boldsymbol{n}_c = 0.286$ kg/s. The inlet and outlet temperatures of the hydrocarbon oil were $T_{hl} = 400$ K and $T_{hr} = 370$ K, respectively. The phenomenological heat transfer coefficient, $\bar{l}_{qq}$, was obtained from Eq. (14). The total entropy production (Σ) was calculated in all cases. The cases were otherwise as follows:

1. Heat exchanger operated at co-current flow at $T_{cl} = 300$ K and $T_{cr} = 350$ K. The area (A_{co}) needed to perform the required heat exchange (Q) was calculated.
2. Heat exchanger operated at counter-current flow at $T_{cl} = 350$ K and $T_{cr} = 300$ K. The area ($A_{counter}$) needed to perform the required heat exchange (Q) was calculated.
3. Heat exchanger operated at $\Delta(1/T)_{\mathrm{opt}}$, given A_{co}, see case 1. The inlet (T_{cl}) and outlet (T_{cr}) temperatures of the cooling water were calculated from Eq. (11). This case is thus an optimized case 1.
4. Heat exchanger operated at $\Delta(1/T)_{\mathrm{opt}}$, given $A_{counter}$, see case 2. The inlet (T_{cl}) and outlet (T_{cr}) temperatures of the cooling water were calculated from Eq. (11). This case is thus an optimized case 2.

In order to solve Eqs. (7) and (8) for all cases, we discretized the integrals

$$Q = \sum_{n=1}^{p} J'_{q,n} A_n, \qquad (17)$$

$$\Sigma = \sum_{n=1}^{p} J'_{q,n} \Delta\left(\frac{1}{T}\right)_n A_n, \qquad (18)$$

$$A = \sum_{n=1}^{p} A_n. \qquad (19)$$

The discretization gives a series of p rectangular control volumes; each of them spans the xy-plane completely. The heat exchange area of one sub-volume (in the yz-plane), is denoted A_n. We chose $p = 10$ in the calculations.

We circumvented the problem that we do not know the temperature profiles as a function of z. Instead, we calculated the temperature profiles corresponding to the fluid's enthalpy. Assuming constant heat capacities within the actual temperature range, we used the following calculation procedure for cases 1 and 2. Dividing the duty (Q) by the number of control volumes, gives $q = Q/p$. The temperature profile of each of the two streams, in *enthalpy space,* is then given by the linear relation

$$T_i(n) = T_{il} + \left(\frac{T_{ir} - T_{il}}{p}\right)\left(n - \frac{1}{2}\right), \quad i = \mathrm{h}, \mathrm{c}, \quad n = 1, 2, \ldots, p, \tag{20}$$

where the subscript i can be c (cold stream) or h (hot stream) and the numbering starts from the left in Fig. 1. The area corresponding to each control volume is found by equating the summand of Eq. (17) to q. By rearranging and substituting $J'_{q,n}$ in the summand with Eq. (5), we obtain

$$A_n = \frac{Q}{pUT_\mathrm{h}^2(n)\left(\frac{1}{T_\mathrm{c}(n)} - \frac{1}{T_\mathrm{h}(n)}\right)}. \tag{21}$$

The calculations in cases 3 and 4 were carried out by a trial and error binary search algorithm. First, the value of $\Delta(1/T)_{\mathrm{opt}}$ was guessed. Then the area needed to perform the heat exchange, given by Q, was calculated using the procedure given above and compared to A_{co} of case 1 or A_{counter} of case 2. If the calculated area was smaller than the target value, the value of $\Delta(1/T)_{\mathrm{opt}}$ was decreased, otherwise it was increased.

A change in the value of p did not alter the results significantly, since the temperature profiles are linear in enthalpy space and U was kept constant; making $\bar{l}_{qq}$ close to constant. The number of steps may, however, not be sufficient if $\bar{l}_{qq}$ is allowed to vary more.

4. Results and discussion

4.1. The reference cases

The results are presented in Table 1. Consider first cases 1 and 2, the co-current and the counter-current modes of operation. We see that the area requirement is 19% less at counter-current than at co-current heat exchange for the same entropy production. Table 1 shows that the thermodynamic driving force ($\Delta(1/T)$) through the heat exchanger is closer to constant in case 2 than in case 1, but it is not completely constant. At the inlet of the heat exchanger it is $3.57 \times 10^{-4}\ \mathrm{K}^{-1}$, at the outlet the force is $6.31 \times 10^{-4}\ \mathrm{K}^{-1}$. Case 2 is therefore better from a second law point of view than case 1. This is nothing but a well-known engineering result.

Cases 1 and 2 serve as references for the next cases. Neither of the cases 1 and 2 have a constant force, and the heat exchange process they represent are therefore not optimal according to the second law.

4.2. The lower bound on the entropy production in heat exchange

Case 3 gives a quantitative measure for how far it is possible to reduce the entropy production rate using the area given by case 1, and therefore gives the lower bound for the entropy production rate of case 1. Case 4 does the same for case 2. The entropy production rate of case 1 can be reduced by 20%, by operating the heat exchanger according to the principle of EoF. For case 2, the entropy production rate can be reduced by 1.9%; lower than for case 1, as expected, since the forces in case 2 are more alike.

We have in the optimum case 3 that the outlet temperature ($T_{\mathrm{cl}} = 346$ K) is lower than the outlet temperature of case 1 ($T_{\mathrm{cr}} = 350$ K). The inlet temperature of case 3 ($T_{\mathrm{cr}} = 324$ K) is considerably higher than that of case 1, ($T_{\mathrm{cl}} = 300$ K). We know that the energy balance is fulfilled for case 1. In case 3, the optimum driving force has been decided, the next step is to find fluids and fluid flows that fulfill the energy balance for the process with the new temperatures. If it is possible to change fluids and operating conditions accordingly, we know that a reduction in the entropy production is obtainable. A change of this sort may alter the factor mC_p, and thus the frictional losses in the heat exchanger. This means that we are in a tradeoff situation. The gain by application of EoF, here the possibility to cool at a higher temperature, must in practice be weighted with an increased cost in pumping and/or a more expensive fluid.

Cases 3 and 4 give corresponding values of area and minimum entropy production rate. We see that

Table 1
Entropy production rate and heat exchange area from cases 1–4[a]

	Σ (W K^{-1})	A (m^2)	T_{cl} (K)	T_{cr} (K)	$\Delta(1/T)_\mathrm{l}$ (K^{-1})	$\Delta(1/T)_\mathrm{r}$ (K^{-1})
Case 1	29.05	3.08	300	350	8.33×10^{-4}	1.54×10^{-4}
Case 2	29.05	2.51	350	300	3.57×10^{-4}	6.31×10^{-4}
Case 3	23.22	3.08	346	324	3.87×10^{-4}	3.87×10^{-4}
Case 4	28.51	2.51	336	315	4.75×10^{-4}	4.75×10^{-4}

[a] The heat exchanger duty (Q) was 60 kW and the heat transfer coefficient (U) was 340 W/m^2 K. The inlet and outlet temperatures of the hot fluid were kept constant at respectively, $T_{\mathrm{hl}} = 400$ K and $T_{\mathrm{hr}} = 370$ K.

the larger the area is, the smaller is the entropy production rate; as expected. The relation between a constant force and its corresponding minimum entropy production rate, gives a family of ideal operating lines. We have called such lines *isoforce operating lines* [8,15]. An isoforce operating line is a line that obeys the condition given by Eq. (11). Each line corresponds to a given duty if the area is given and vice versa.

We can find an arbitrary isoforce line directly starting from two inlet temperatures. The corresponding area can next be found from enthalpy considerations. If the boundary conditions of case 4 were realizable, we would be able to save 19% on the area in case 1 in this manner, and still carry out the cooling in question (possible frictional losses not considered).

We have seen from the example above that the phenomenological coefficient $\bar{l}_{qq}$ need not be constant in a valid application of the principle of EoF. Prigogine [13], see e.g., [16], showed that a system in its stationary state has minimum entropy production when the fluxes are linear in the forces and the coefficients are constant. We are speaking of a different situation. Prigogine [13] found fully integrated equations of transfer with constant coefficients, as a result of a variational principle, while we use flux equations on a partially integrated level and find how the production (the heat transfer) should be distributed, in the most energy efficient way, see [7] for further details.

The value of the coefficient decides the value of the force, as seen from Eq. (12). By increasing the local coefficient, the area can be reduced. The type of flux decides the form of the driving force, because the product of flux and force must define the entropy production. The force may therefore be different for a cooling machine, [1,9], and a heat exchanger. When $\Delta T \gg T$, the thermal driving force can be approximated with ΔT [5].

5. Conclusion

We have shown, using entropy production minimization, why it is more advantageous, from the point of the second law, to operate a heat exchanger in counter-current than in co-current mode. We have seen by analyzing a numerical example, that a family of operating lines called isoforce operating lines may be used to assess the efficiency of heat exchangers. Area reductions can be obtained by replacing normal operation by isoforce operation, but these savings must be traded off by frictional losses or material costs [3]. The operation that gives minimum entropy production for parallel transport paths, namely constant force operation, serves as an ideal target for energy efficiency in the heat exchange process. In order to benefit from the principle of EoF, it may be interesting in the future to fit experimental results for heat transfer to the flux equation that uses $\bar{l}_{qq}$ instead of the integrated Fourier's law that uses U.

Acknowledgements

Lars Nummedal is grateful for a grant given by The Norwegian Research Council. Dick Bedeaux is thanked for help in clarifying the mathematics.

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